

# An Alternative Methodology Based on Spectral Analysis for a Direct Access to Ring Resonator Parameters

Yaneck Gottesman, Dominik G. Rabus, *Senior Member, IEEE*, Elchuri V. Krishna Rao, and Badr-Eddine Benkelfat, *Senior Member, IEEE*

**Abstract**—An alternative method of ring resonators (RRs) spectral analysis is proposed and demonstrated here. Unlike earlier procedures, the phase information is extracted here from pure spectral intensity data to examine the RR signatures in the time domain. This procedure gives a direct access to all resonator parameters. Also, it is well suited for an automated assessment of dispersive properties of rings in a wide spectral domain.

**Index Terms**—Characterization, integrated optics, phase reconstruction, ring resonators (RRs).

## I. INTRODUCTION

SINCE their advent a few decades ago, ring resonators (RRs) have undoubtedly played a major role as key components in the field of optical communications [1], [2]. Also, their recent introduction into the field of bio-photonics further revealed their high potential as base components for the realization of compact biosensors to detect and identify their surface-functionalized molecules [3], [4]. Hence, it is necessary to develop highly sensible and efficient methods that give a direct access to all resonator parameters (including dispersive properties) and further help to detect minute modifications in their properties, particularly when employed as biosensors. In this context and in contrast to the existing procedures, we propose here an alternate method to analyze RR from pure spectral intensity data. It consists in reconstructing *phase information* that provides RR signature in the time domain and ultimately gives a direct access to all RR characteristics. Also, as shown here, this method is particularly well suited for automation.

## II. SPECTRAL ANALYSIS AND CONVENTIONAL METHODS

To date, the method often employed to characterize a RR is based on spectral measurements in transmission mode recorded using a fine tunable laser. Subsequently, the RR parameters, i.e., the coupling coefficient  $\kappa$ , the global propagation losses in the ring  $\alpha$ , and the optical length  $D$ , can be extracted from spectral

characteristics (minimum and maximum of transmitted power as well as the spectrum half-width at half-maximum), as detailed in [5]. Also, these are extracted quite often by fitting an analytical model with the transmission data around a desired wavelength  $\lambda$  [6]. Although efficient, these methods do not permit to probe into the formation of the interference pattern for the following reasons: 1) the phase information  $\phi$  is not available explicitly and 2) usually, a single free spectral range (FSR) of RR is exploited to extract the parameters at a given  $\lambda$  [5]. As a result, the temporal resolution accessible by such spectral window is limited. In this regard and as shown later, the reconstruction of missing phase information over a broad spectral region constitutes the key element of the proposed methodology. This helps to convert the data in the spectral domain to the time domain and further gives an insight into the understanding of the recorded interference pattern. By definition, such a conversion is obtained through Fourier transformation of the resulting complex spectrum. This permits to obtain temporal information quite similar to other sophisticated methods such as ring-down or nonlinear optical up-conversion previously used for spherical cavities [7].

## III. PHASE RECONSTRUCTION AND TEMPORAL SIGNATURE

Let us denote  $h(\nu)$  as the complex amplitude of RR transmission and  $h(t)$  as its Fourier transform (or impulse response), with  $\nu$  the optical frequency and  $t$  the time variable. Following [8], the Laplace transform  $L(p)$  of RR impulse response is

$$L(p) = (-\alpha + \tau e^{-Tp})(-\alpha\tau^* + e^{-Tp})^{-1} \quad (1)$$

where  $p$  is the Laplace variable,  $\tau$  the transmission coefficient of the coupler,  $T$  the light time of flight in the ring, and  $*$  the complex conjugation (with  $|\tau|^2 + |\kappa|^2 = 1$  for a lossless coupler with a coupling coefficient of  $\kappa$ ). A rapid examination of (1) reveals that  $L$  has only one zero at  $p_0 = -T^{-1} \ln(\alpha\tau^{-1})$ . The RRs are generally designed to favor sharp  $\lambda$ -resonances (and longer photon lifetimes in the ring) to attain the so-called critical coupling conditions (i.e.,  $\alpha = |\tau|$ ). However, in all practical cases,  $\alpha$  is lower than or equal to  $|\tau|$ . Hence,  $p_0$  is located in the right-hand side (RHS) of the complex plane. Consequently, as detailed in [9], the RR filter belongs to the class of “minimal phase” filters. A useful property of these filters can then be beneficially exploited, namely the relationship between  $-\ln(|h(\nu)|)$  and  $\phi(\nu)$  through Hilbert transform ( $H_T$ ). This means the missing phase information can be reconstructed

Manuscript received March 30, 2009; revised May 19, 2009. First published June 30, 2009; current version published September 18, 2009.

Y. Gottesman and B.-E. Benkelfat are with the Institut TELECOM-T&M SudParis, Laboratoire, SAMOVAR CNRS-INT UMR 5157, 91011 Evry, France (e-mail: yaneck.gottesman@it-sudparis.eu; badr-eddine.benkelfat@it-sudparis.eu).

D. G. Rabus is with the Baskin School of Engineering, University of California at Santa Cruz, Santa Cruz, CA 95064 USA (e-mail: rabus@iee.org).

E. V. K. Rao is with the Centre National de la Recherche Scientifique (CNRS) Laboratoire de Photonique et de Nanostructures (LPN), 91460 Marcoussis, France (e-mail: elchuri.rao@lpn.cnrs.fr).

Digital Object Identifier 10.1109/LPT.2009.2025603

from the intensity of the transmission spectrum  $S(\nu) = |h(\nu)|^2$  using the relation

$$\phi(\nu) = H_T \left( -\ln \sqrt{S(\nu)} \right). \quad (2)$$

Once  $\phi$  is evaluated,  $h(t)$  can then be calculated through the Fourier Transform of the resulting complex spectrum ( $h(\nu) = |h(\nu)| \exp(i\phi(\nu))$ ). A further exploitation of this information is carried out by calculating the expected theoretical response of RR in the time domain [10] by performing Fourier Transform of (1). This is realized by replacing the Laplace variable  $p$  with  $i2\pi\nu$  [since  $h(\nu) = L(i2\pi\nu)$ ] and further rewriting (1) as

$$h(p) = t + (t^2 - 1)t^{*-1} \sum_{n=1}^{+\infty} (\alpha t^* e^{iTp})^n. \quad (3)$$

One can clearly deduce from the Fourier transform of (3) that  $h(t)$  is composed of evenly spaced  $\delta$ -functions representing the RR signature in the time domain. The first (say peak  $P_1$ ) is related to the light impulse that passes directly through the coupler while all the other folding peaks ( $P_j$  with  $j > 1$ ) correspond to the light that traveled  $j$  loops in the ring. The intensity of the peaks [deduced from (3)] is

$$P_1 = 1 - \kappa \quad \text{and} \quad P_j = (1 - \kappa)^{j-1} k^2 \alpha^{2j-1}. \quad (4)$$

Obviously, they contain all the information at the origin of the interference pattern visible in the spectrum. The coupler coefficient as well as the ring propagation losses can then be directly assessed by solving the set of equations represented by (4). Due to the fact that  $P_j/P_{j+1}$  (with  $j > 1$ ) is a constant, the relative intensities of the first three peaks,  $P_1/P_2$  and  $P_2/P_3$ , are alone needed for a complete assessment of RR. This means neither their absolute values nor any exploitation of other peaks is necessary. Furthermore, quite interestingly, one need not take into consideration the insertion losses in the component since the relative intensities are exploited alone.

However, in the framework of the methodology described before, the resolution of different peaks in the time domain imposes some adequate experimental conditions on the width of the spectral window ( $\Delta\nu$ ) employed to record the transmitted spectrum. More precisely, in agreement with the Heisenberg's uncertainty principle, we deduce that  $\Delta\nu$  must be at least larger than twice the RR-FSR (for a Gaussian line shape window).

#### IV. RESULTS AND DISCUSSION: DIRECT ASSESSMENT OF RRs

Fig. 1(a) depicts the experimental transmitted spectrum  $S(\nu)$  of an InP-based RR whose fabrication details are given in [2]. This is recorded for TE polarization by sampling 40 points per FSR over a broadband window of 60 nm using a fine tunable laser. As detailed in Section III, a direct assessment of RRs further requires the exploitation of their signature in the time domain subsequent to phase reconstruction. This is carried out as follows: 1) the sampled spectral data are filtered by multiplying  $S(\nu)$  with a chosen Gaussian function  $G$  arbitrarily centered at  $\lambda_c = 1545.5$  nm [see dashed region in Fig. 1(a)] not only to access RR parameters at  $\lambda_c$  but also to obtain Gaussian line shapes for  $\delta$ -functions in the time domain. Here, the FWHM of  $G$  is so chosen (six times the RR-FSR) as to offer a good tradeoff

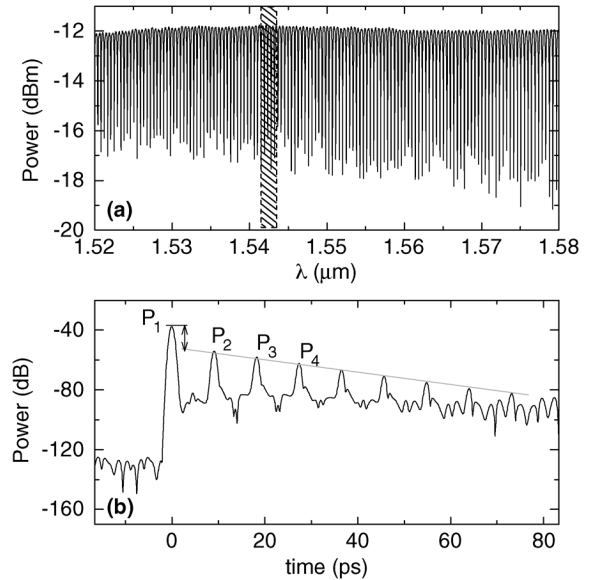


Fig. 1. Procedure for a direct assessment of RR parameters. (a) Transmission spectrum recorded with 40 points per FSR. Note the evolution of fringe visibility with  $\lambda$ . (b) Time-domain signature of RR after applying the proposed methodology over a limited spectral window [see dashed box in (a)].

between temporal resolution and wavelength selectivity; 2) the missing phase information is then reconstructed from (2) by evaluating the Hilbert transform  $H_T$  of  $\sqrt{S}G$ . Here,  $H_T$  is numerically calculated following the recommendations detailed in [9] and further using the Wiener-Lee transform as in [11]; 3) the complex transmission and the RR signature in the time domain are then obtained after calculating the Fourier transform of  $\sqrt{S}G \exp(i\phi)$  [using (2)]. The resulting temporal signature, as seen from Fig. 1(b), exhibits in log scale several evenly spaced peaks with a regular decrease in their amplitude. This indeed ensures that the analyzed signature corresponds to a theoretical RR temporal signature; and 4) finally, the magnitude  $|h(t)|$  of the first three peaks ( $P_1$ ,  $P_2$  and  $P_3$ ) is then exploited by solving (4) for a direct evaluation of all RR parameters.

The parameters thus extracted at  $\lambda_c$  from the first three peaks are  $\kappa = -6.9$  dB and  $\alpha = -3.1$  dB. Using these values, we have simulated the transmission spectrum and compared with the one experimentally recorded under high sampling (100 points per FSR) conditions [see Fig. 2(a)]. The close agreement between these spectra (comprising phase and intensity) clearly confirm the exactitude of these values and thereby validate the proposed methodology. Also, additional simulations (not detailed here) clearly reveal that the phase is highly sensitive to changes in the refractive index of  $\Delta n \sim 10^{-4}$  to  $10^{-5}$  and could be used to probe biosensors.

The fact that the RR is analyzed in the time domain presents some advantages: 1) its temporal signature is unaffected by any additive Gaussian noise (due to, e.g., light injection instability or photodiode noise) encountered during spectral measurements as confirmed through simulations and 2) the peaks maxima can be easily localized using a simple algorithm (based on sign change of the first derivative). This means the assessment of RR characteristics can be easily realized over a broadband window through an automated analysis of all peaks

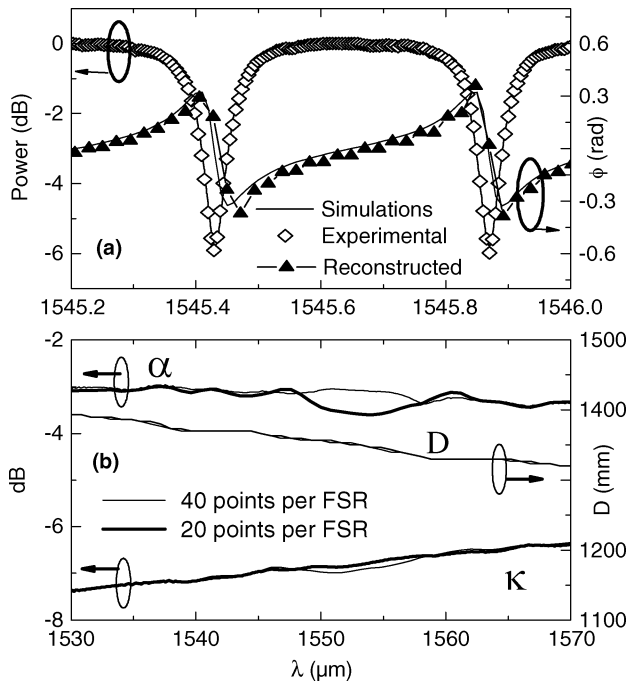


Fig. 2. Confirmation of extracted RR parameters. (a) Comparison of simulated (using  $\kappa = -6.9$  dB and  $\alpha = -3.1$  dB) and measured (high sampling conditions) complex transmission spectra with phase information. (b) Dispersive properties of RR after the application of CTFA (see text).

( $P_1, \dots, P_n$ ). It further confirms that the proposed methodology is also valid for an arbitrarily selected spectral window centered at any wavelength ( $\lambda_c$ ). The latter is accomplished by performing concurrent time–frequency analysis (CTFA) [12], [13]. This consists in filtering  $S(\nu)$  by sliding the Gaussian function  $G$  across the global spectral window and repeating the methodology for all filter positions.

The resulting signals are then automatically analyzed in the time domain to obtain the dependence of researched parameters on the central wavelength of  $G$ . Fig. 2(b) depicts the resulting dispersive properties of  $\alpha$ ,  $\kappa$ , and also the optical length  $D$  evaluated from the separation of  $P_1$  and  $P_2$  directly related to the effective group refractive index. They reveal that  $\alpha$  is almost independent of  $\lambda$  with fluctuations below 0.3 dB. In contrast,  $\kappa$  and  $D$ , respectively, exhibit variations of  $\sim 1$  dB and 5% over 40 nm. We note here that these results are compliant with the visible evolution of the interference contrast across the spectrum [see Fig. 1(a)] and the chirp in the position of RR resonant frequencies. The evolution of  $\alpha$  and  $\kappa$  is tentatively explained as follows. The race-track-shaped RR employed here is designed to favor a long coupling length between the straight guide and RR. Since the optimal coupling length depends on  $\lambda$ , some dispersive effects can be expected in the coupler depending on the type of coupling mechanism. On the other hand, since the guides are deeply etched, the losses (due to propagation and RR bends) can only be expected to depend slightly on  $\lambda$ . However, more work is in progress to identify the physical origin of such dispersive properties.

Another flexibility offered by the time-domain analysis employed here is worth mentioning. It concerns a possible reduction in sampling conditions of the transmission spectrum. This is experimentally determined by finding out a minimum number of points per FSR that ensure the expected theoretical RR signature in the time domain. For example, we have plotted in Fig. 2(b) (see the light curve) the experimental dispersion of RR parameters obtained using only 20 points per FSR. The close agreement between these curves (20 versus 40 points per FSR) clearly confirms that one can reduce the measurement time, and hence speed up the assessment of RR.

## V. CONCLUSION

As the prime objective of this letter, we have proposed and demonstrated the efficiency of an alternate method of spectral analysis to completely assess RRs. We further showed that it is well suited for an automated characterization of dispersive properties of RRs. Lastly, it is worth mentioning that simulations show that the phase is highly sensitive to effective index changes (up to  $\Delta n \sim 10^{-5}$ ) as needed for biophotonic applications.

## REFERENCES

- [1] B. E. Little, S. T. Chu, H. A. Haus, J. Foresi, and J.-P. Laine, "Microring resonator channel dropping filters," *J. Lightw. Technol.*, vol. 15, no. 6, pp. 998–1005, Jun. 1997.
- [2] D. G. Rabus, M. Hamacher, U. Troppenz, and H. Heidrich, "Optical filters based on ring resonators with integrated semiconductor optical amplifiers in GaInAsP–InP," *IEEE J. Sel. Topics Quantum Electron.*, vol. 8, no. 6, pp. 1405–1411, Nov./Dec. 2002.
- [3] A. Armani, R. Kulkarni, S. Fraser, R. Flagan, and K. Vahala, "Label-free, single molecule detection with optical microcavities," *Science*, vol. 317, pp. 783–787, Aug. 2007.
- [4] K. De Vos, I. Bartolozzi, E. Schacht, P. Bienstman, and R. Baets, "Silicon-on-insulator microring resonator for sensitive and label-free biosensing," *Opt. Express*, vol. 15, no. 12, pp. 7610–7615, 2007.
- [5] J. Scheuer and A. Yariv, "Fabrication and characterization of low-loss polymeric waveguides and micro-resonators," *J. Eur. Opt. Soc.*, vol. 1, pp. 06007-1–06007-5, 2006.
- [6] D. Rabus, *Integrated Ring Resonators—The Compendium*. Berlin, Germany: Springer-Verlag, 2007.
- [7] T. Siebert, O. Sbarski, M. Schmitt, V. Engel, W. Kiefer, and J. Popp, "The mechanism of light storage in spherical microcavities explored on a femtosecond time scale," *Opt. Commun.*, vol. 216, pp. 321–327, 2003.
- [8] A. Yariv, "Universal relations for coupling of optical power between micro-resonators and dielectric waveguides," *Inst. Electr. Eng. Electron. Lett.*, vol. 36, no. 4, pp. 321–322, Feb. 2000.
- [9] A. Papoulis, *The Fourier Integral and Its Applications*. New York: McGraw-Hill, 1962.
- [10] Y. Gottesman, E. V. K. Rao, and D. Rabus, "New methodology to evaluate the performance of ring resonators using optical low-coherence reflectometry," *J. Lightw. Technol.*, vol. 22, no. 6, pp. 1566–1572, Jun. 2004.
- [11] A. Carballar and M. A. Muriel, "Phase reconstruction from reflectivity in fiber Bragg gratings," *J. Lightw. Technol.*, vol. 15, no. 8, pp. 1314–1322, Aug. 1997.
- [12] Y. Gottesman, A. Parini, E. V. K. Rao, and B.-E. Benkelfat, "Benefits of concurrent spatial and spectral analysis of photonic components," presented at the IEEE Indium Phosphide Related Mater. Conf. (IPRM 2008), Versailles, France, May 25–29, 2008, Paper WeP49.
- [13] A. Parini, P. Hamel, A. De Rossi, S. Combrié, N.-V.-K. Tran, Y. Gottesman, R. Gabet, Y. Jaouen, and G. Vadalà, "Time-wavelength reflectance maps of photonic crystal waveguides: A new view on disorder-induced scattering," *J. Lightw. Technol.*, vol. 26, no. 23, pp. 3794–3802, Dec. 1, 2008.